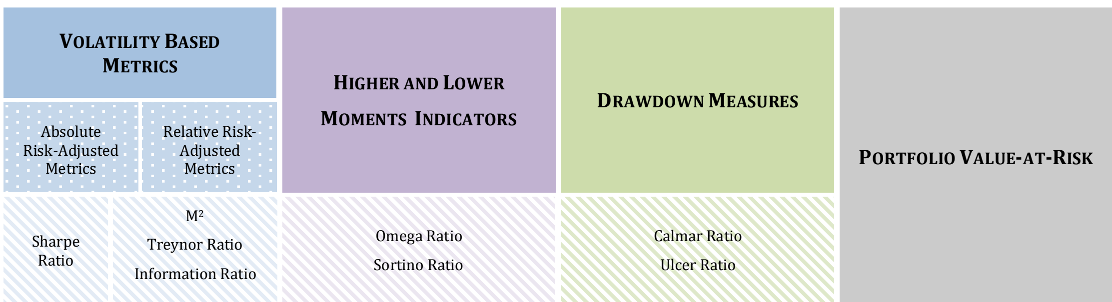
Investors often rely on Risk-adjusted performance measures such as the Sharpe ratio to choose appropriate investments and understand past performances. The purpose of this paper is getting through a selection of indicators (i.e. Calmar ratio, Sortino ratio, Omega ratio, etc.) while stressing out the main weaknesses and strengths of those measures.

1. Introduction

Investors and analysts interested in measuring how good is an investment clearly cannot only rely on the absolute returns provided by the asset, or compare those returns to a benchmark, but must also consider the Risk they bear holding this investment, *i.e.* how much returns the investment has generated relative to the amount at Risk it has taken over a period of time.

There are plenty of alternative Risk-adjusted performance measures, and financial literature has widely discussed this topic, the purpose of this paper is not getting through all of them, but the major ones. Please note that returns, standard deviations, and other metrics can be computed on any period, it is nevertheless advised to annualise figures to make the comparison easier.

One decided to structure and to present those indicators as follows:



2. The Volatility-Based Metrics

Financial analysts frequently use volatility-based metrics to assess portfolio performances. Paradoxically, despite that those measures tend to be very intuitive they remain difficult to interpret; it can be because of the absence of benchmark (*cf.* Absolute-Risk Adjusted Metrics Section) or the problem to interpret the relative performances of a portfolio to a benchmark (*cf.* Relative-Risk Adjusted Metrics Section). In addition, most volatility-based metrics rely on historical data and on the assumption that returns are normally distributed, undermining the impact of peak-to-valley or downside Risk.

2.1. Absolute-Risk Adjusted Metrics

Those metrics evaluate the portfolio risk-adjusted performances without considering any benchmark.

2.1.1. The Sharpe Ratio

Introducing the notion of *Reward-to-Variability* in the Journal of Business (1966), William Sharpe set up milestones of the performance assessment of Mutual Funds. The ratio measures the excess return of a portfolio to the Risk free rate to the standard deviation of the portfolio. In a Risk adverse framework, investors are looking for a high ratio.

$$Sharpe\ Ratio = \frac{R_P - R_F}{\sigma_P}$$

R_P = Portfolio return (annualised)

R_F = Risk free rate (annualised)

σ_P = Standard deviation of the daily portfolio returns

2.1. Relative-Risk Adjusted Metrics

Those metrics evaluate the portfolio risk-adjusted performances while considering a benchmark. One of the major weaknesses relies in the benchmark selection and on the interpretation of the metric. You can easily identify if your portfolio outperformed or underperformed the index, but you have no clue on how the performance was achieved.

2.1.1. The Modigliani-Modigliani Measure (“M^2”)

Modigliani-Modigliani (1997) reconcile the notion of benchmark and Sharpe ratio in a single formula, and find that the portfolio and its benchmark must have the same Risk to be compared. The idea is to lever or delever a portfolio so that its standard deviation is identical to that of the market portfolio. The M^2 of the portfolio is the return of this portfolio that can be compared to the market return.

$$M^2 = R_P + Sharpe\ Ratio * (\sigma_B - \sigma_P)$$

R_P = Portfolio return (annualised)

σ_B = Standard deviation of the daily benchmark returns

σ_P = Standard deviation of the daily portfolio returns

2.1.2. The Treynor Ratio

The metric - developed by Treynor (1965) - measures the relationship between the excess return of a portfolio on the Risk free rate to the Beta of the portfolio. The ratio is particularly adequate to assess performances of extremely diversified portfolios; this is why it is largely used by academics. In practice, investment analysts cannot ignore the Idiosyncratic Risk; this is why the ratio is much less used than the Sharpe one.

$$Treynor\ Ratio = \frac{R_P - R_F}{\beta_P}$$

R_P = Portfolio return (annualised)

R_F = Risk free rate (annualised)

β_P = Systemic Risk, Risk than cannot eliminated by diversifying the portfolio

2.1.3. The Information Ratio

The Information ratio indicates how much excess return is generated from the amount of excess Risk taken relative to the benchmark. This indicator is appreciated by investors that can easily compare performances of their portfolio to a benchmark and set investment objective to the investment manager.

$$Information\ Ratio = \frac{R_P - R_B}{\theta_P - \theta_B}$$

R_P = Portfolio return (annualised)

R_B = Benchmark return (annualised)

σ_P = Standard deviation of the daily portfolio returns

σ_B = Standard deviation of the daily benchmark returns

3. Higher and Lower Moments Indicators

In the above Section, one only considered the first moment (mean of returns), and second moment of the distribution (standard deviation of returns); when actually the third and fourth moments must also be considered (*i.e.* Skewness and Kurtosis, both indicate the shape of the distribution). Any Risk adverse investor would be interested by (1) high returns, (2) a low volatility, (3) more positive returns than negative ones (distribution right skewed), and (4) a leptokurtic distribution (not too many extreme returns, then thin tails).

Risk-adjusted moments measures rely most of the time on a single threshold, a minimal return level chosen by investors; it is important to precise that the selection of the latter could lead to distorted interpretations of the metric. In addition, if parameters are derived using a parametric approach, one face the same weaknesses that those specified for volatility based metrics (*i.e.* normalised returns, *etc.*).

3.1. The Omega Ratio

Shadwick and Keating (2002) elaborate a gain-loss ratio capturing the higher moments in the returns distribution, the Omega ratio. The authors split the returns distribution above and below a minimum acceptable return for investors and assess the probability of occurrence above and below the partitioning. The higher the metric, the better the investment.

$$\Omega\ Ratio = \frac{\int_{R_{TI}}^{b} 1 - F(R_i)dx}{\int_{a}^{R_{TI}} F(R_i)dx} = \frac{Upside\ Potential}{Downside\ Potential}$$

R_i = Daily returns

R_{TI} = Minimal acceptable daily return

[a,b] = Interval of possible returns

3.2. *The Sortino Ratio*

One already mentioned that the Sharpe ratio does not tell us anything about the returns distribution, is the distribution of returns symmetric around the mean, or mainly driven by returns above the mean (upside potential) or below the mean (downside potential). A natural extension of the Sharpe ratio is the Sortino indicator only using the downside Risk as a denominator and replacing the Risk-free rate by the minimal acceptable return.

$$Sortino\ Ratio = \frac{R_P - R_T}{\theta_D}$$

R_P = Portfolio return (annualised)

R_T = Minimal acceptable portfolio return

σ_D = Standard deviation of the daily portfolio returns below the minimal acceptable return

3.3. *The Upside Potential Ratio*

The Upside Potential ratio - developed by Sortino, Van der Meer and Plantinga (1999) - is a hybrid version between the Omega ratio and the Sortino ratio. The indicator has the upside potential for numerator (as for the Omega ratio), and the downside Risk for denominator (as for the Sortino ratio).

$$Upside\ Potential\ Ratio = \frac{\int_{R_{TI}}^{b} 1 - F(R_i)dx}{\theta_D} = \frac{Upside\ Potential}{Downside\ Risk}$$

R_i = Daily returns

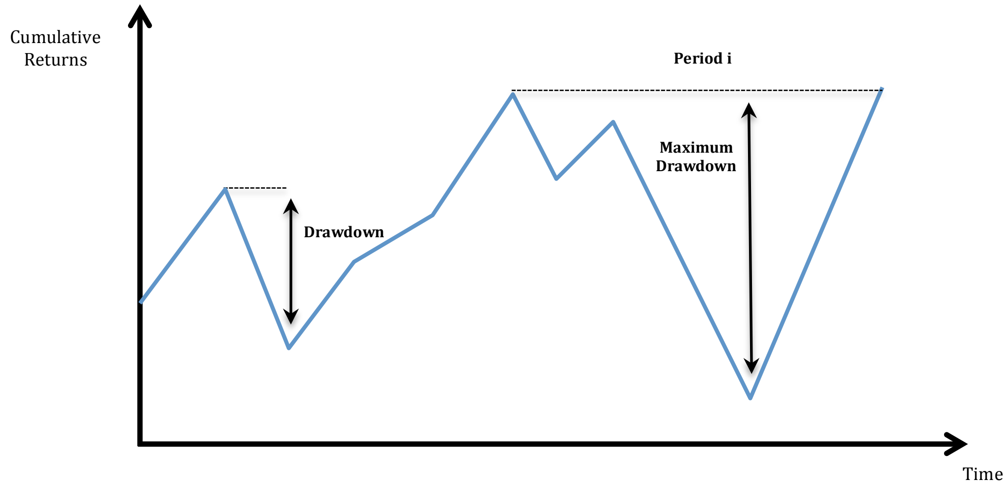
R_{TI} = Minimal acceptable daily return

[a,b] = Interval of possible returns

σ_D = Standard deviation of the daily portfolio returns below the minimal acceptable return

1. Drawdowns Measures

Intuitive Risk-adjusted measures may be derived from past drawdowns of the returns distribution. Drawdowns consist of focusing on peak-to-valley declines during a specific period of investment; usually the percentage between the peak and the subsequent though. The maximum drawdown may be defined as the maximum loss from a peak-to-valley over the specified period. Drawdowns are very sensitive to the frequency of the measurement interval and to the length of the sample; assets measured on a daily interval will tend to have higher drawdowns than assets measured on a monthly interval; in addition managers with a large historical dataset will tend to have larger drawdowns.



1.1. The Calmar Ratio

The Calmar ratio developed by Terry Young (1991) is a measure very frequently used in the Hedge Fund industry and in the context of both illiquid assets and lack of historical data. The Calmar ratio shares the same numerator as the Sharpe ratio, but gets for denominator the maximum drawdown over the period (*i.e.* 1 year).

$$Calmar\ Ratio = \frac{R_P - R_F}{Max\ D}$$

R_P = Portfolio return (annualised)

R_F = Risk free rate (annualised)

Max D = Maximum Drawdown over the period

1.2. The Ulcer Ratio

Peter G. Martin (1987) developed the Ulcer index with the purpose of not only considering the depth of the price decline but also its duration. The index measures the negative return of each period below the previous peak or watermark. Consequently, deep and long lasting drawdowns will have a significant impact on the ratio.

$$Ulcer\ Ratio = \sqrt{\sum_{i=1}^{n} \frac{{D_i}^2}{n}}$$

D_i = Drawdown since the last peak in the period

n = Number of observations/days in the period

1. Portfolio Value-at-Risk

Another approach would consist in assessing the loss of the portfolio using a Value-at-Risk method. The different methods to compute the VaR (*i.e.* Historical, Parametric, Monte-Carlo) have already been developed in another Section of this website (Quantik.org - "Download Center", "VaR Models (Parametric, Monte-Carlo)").

2. Conclusion

If the universe of Risk-adjusted performance measures is very large and still continues to broaden, this paper was dedicated to highlight the main indicators that frequently are used in the industry. We also expected to stress out the main weaknesses associated to the different approaches.

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