



This working paper is dedicated to investigating if a positive Gamma strategy can be interpreted in the same way as having a positive Convexity strategy. Despite that the first strategy is usually applied to equity derivatives and the second one to credit instruments, we expect to demonstrate that both structures can be understood in the same way.

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1. Introduction

Keeping positive Gamma or positive Convexity strategies can veritably be puzzling for Traders, and can rapidly lead them to face significant losses if no measures are taken to maintaining that positive framework.

Keeping long story short, positive Gamma or positive Convexity can be understood as being Long when markets are bullish (or sell high for hedging purposes), or being Short when markets are bearish (buy low for hedging purposes). In the context of a neutral strategy, such a structure would then be optimal since traders would see their positions appreciating both when markets are going up or down.

To demonstrate if there is an analogy between positive Gamma and positive Convexity, we organise this paper as follows:

- First Section: Section dedicated to brush up the reader knowledge about Gamma and Convexity ;
- Second Section: Section committed to demonstrate why having a Long positive Gamma strategy or Long positive Convexity strategy can be valuable for traders ;
- Third Section: Section dedicated to exhibit why negative Convexity can be problematic for traders and negatively impact the P&L ;
- Fourth Section: Concise Section devoted to the conclusion.

2. Defining the Gamma and the Convexity?

This Section is dedicated to clarify how the Gamma and the Convexity are computed, and the intuition behind figures. Please note that we will only focus on two types of securities within that paper, equity options for the Gamma indicator, and bonds for the Convexity.

2.1. Understanding the Gamma

Holding an option is equivalent to holding a position in the underlying market; for instance being Long on an European Call means that you have the right to purchase an asset at a given price and time, and that the position will then appreciate when the underlying price of the asset increases.



Since the value of the option is affected by the underlying price of the security, it is good to capture the sensitivity of the option to a change in the underlying price; this is the option Delta and this is represented by:

$$\Delta = \frac{\partial \text{Option Price}}{\partial \text{Underlying Price}} = \frac{\partial V}{\partial S}$$

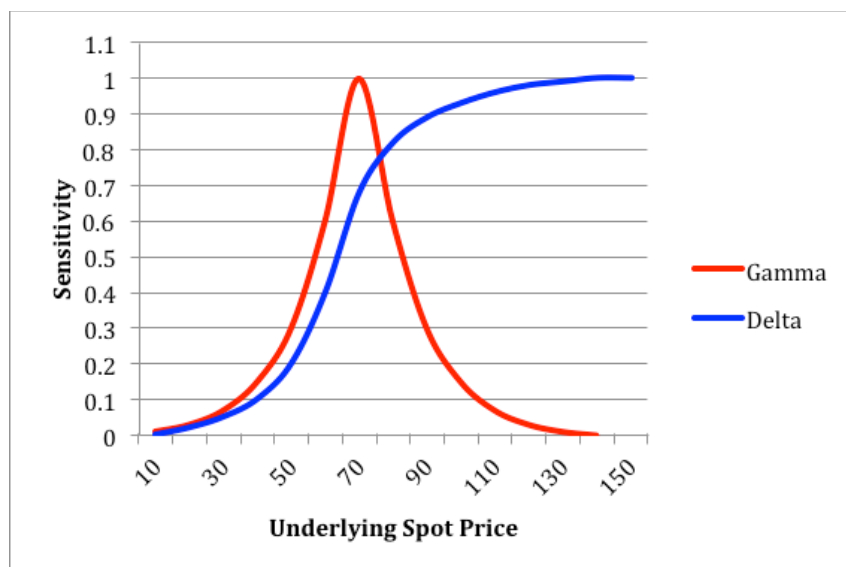
The Delta is positive when you purchase a Call and negative when you purchase a Put. When entirely at the money, the Delta of the Option equals “0.5”, when deeply in the money it tends toward “1”, when significantly out of the money, the delta tends toward “0”.

The Gamma measures the Delta sensitivity to a change of one unit in the underlying asset. A basic and intuitive illustration is considering the option as a racing car, the Delta as the speed at which the car drives and the Gamma as the acceleration or deceleration of the vehicle. The Gamma is represented by:

$$\gamma = \frac{\partial^2 V}{\partial S^2}$$

The Gamma is a highly useful metric when considering a Delta-neutral portfolio, because it evaluates how often a portfolio needs to be re-hedged to keeping the neutral exposure. Back to our above example, imagine that the racing car is so overweighted that it cannot move anymore at a given speed [drifting car]; the Gamma would represent the complementary weights you should add to maintain a perfect immobility of the car if you decide to further accelerate.

The Gamma tends to be higher when the option is at the money, and lower when the option is deeply out or in the money. The chart below illustrates the relationship between the Delta and the Gamma.



2.2. Understanding the Convexity?

Duration and Convexity are sensitivity factors impacted by a parallel shift in the term structure of interest rates. The Duration - or Macaulay Duration - represents the number of years required so that



an investor gets back its money. The Modified Duration represents by how much the price of a fixed income security is impacted by a change in interest rates.

The intuition is then:

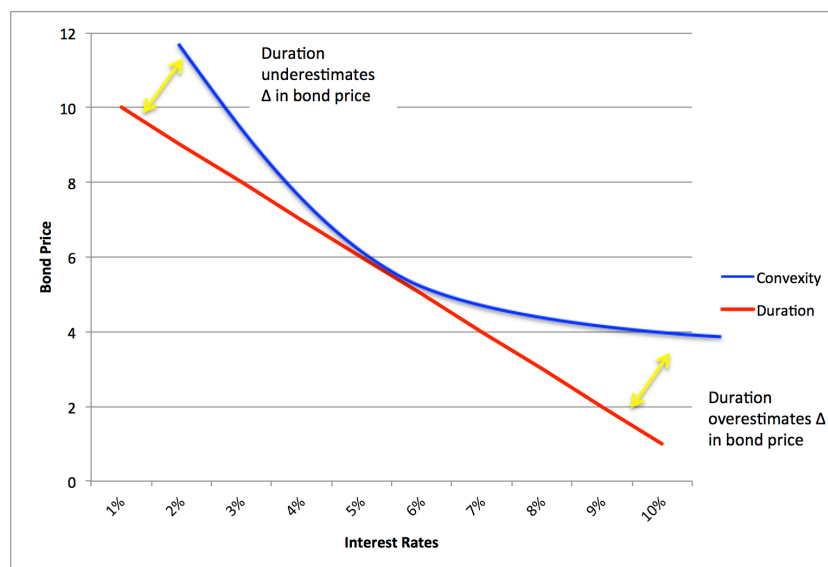
$$\frac{\Delta \text{Price}}{\text{Price}} = -\text{Modified Duration} * \frac{\Delta \text{Interest Rates}}{\text{Interest Rates}}$$

The Duration depends on the periodic cash flows of the security, and it can be concluded - on average - that the higher the coupon rates the lower the Duration, and the higher the maturity of the bond, the higher the Duration [because the reimbursement of the principal plus the last coupon occurrence is far].

A shortfall of the Duration is considering the relationship between the bond price and interest rates to be linear when actually it is not; the Convexity deals with that concern by measuring by how much the Duration is impacted by a change of one unit in interest rates [e.g. basis point].

$$C = \frac{1}{B} * \frac{D^2(B(r))}{Dr^2}$$

When considering positive Convexity, Duration does overestimate the change in the bond price for a high level of interest rates and does underestimate it when interest rates tend to be low. The chart below illustrates that phenomenon.



3. Why Having Positive Gamma or Positive Convexity is Profitable

Having positive Gamma or positive Convexity allows traders to generate positive P&L even with perfectly initially hedged portfolios; this is what we are going to demonstrate within this Section.



3.1. Positive Gamma Impact on P&L

Let's take a look at a very simplistic example; suppose that you are Long positive Gamma via a Call; your Delta at the position level equals "1" [then if the underlying price increases of 1 EUR, your option will also appreciate of 1 EUR]. You initially adopted a Delta neutral strategy by buying 1 Call and selling 1 stock; this seems to be a perfect hedge because if the spot price of the stock increases of 1 EUR, you are gaining 1 EUR on your Call and losing 1 EUR on your stock.

Imagine now that - because of the positive Convexity - when the stock price increases the Delta of your position also increases and reaches "1.1" [this means that the Call appreciates of 1.1 EUR when the stock appreciates of 1 EUR], **when markets are bullish, you are then Long** [you actually earn 0.10 EUR per appreciation of 1 EUR of the stock, because you gain 1.1 EUR on your Call and lose 1 EUR by being Short on the stock]. If you want to keep a Delta neutral portfolio, you then either have to reduce your exposure to the Call or increase your Short exposure; this is **selling when prices are high**.

The intuition is the same if you buy a Put and you buy the underlying stock to keep a Delta neutral portfolio. You will observe that if the spot price decreases, the value of your Put will proportionally appreciate more than the value of the underlying stock in portfolio will depreciate. This proves that when markets are **bearish, you tend to be Short** [having a Put in portfolio represents a Short position in the underlying market], and that if you want to keep a Delta neutral portfolio, you have to buy more of the underlying, or reduce you Short exposure; **this is then buying when prices are low**.

Consequently, positive Gamma strategy generates profits both when markets are going up or down.

3.2. Positive Convexity Impact on P&L

Again, one starts with a basic example to understand how having a positive Convexity can be profitable for traders. Imagine that you initially hold a portfolio of one Long bond and one Short bond; you do want to keep a Duration neutral portfolio [you don't want that your portfolio becomes more or less sensitive to change in interest rates]. Following a downward parallel shift of the yield curve [interest rates decrease at any maturity], the price of your Long position appreciates; the positive Convexity structure claims that - at this interest rate level - the sensitivity of the Long position is actually higher than what the Duration assesses; the bond appreciation is then higher than assessed by the Duration; this is still **being Long when markets are bullish**. In addition, since the initial portfolio is no more Duration Neutral, you either have to decrease you Long exposure [more sensitive to change in interest rates than expected] or increase the Short one to get it neutral again; **this is selling when prices are high**.

In case of an upward parallel shift of interest rates, the effect will exactly be the opposite; the sensitivity of the portfolio is lower than what the Duration measures; **you tend to be Short when markets are bearish**. To maintain the Duration neutral strategy, you have to increase the portfolio sensibility by increasing your Long exposure or decreasing the Short one; **this is buying when prices are low**.

As for the positive Gamma strategy, we see that a positive Convexity structure can then be profitable both when market prices are going up or down.



3.3. Comparative Table Between Positive Gamma and Positive Convexity

This table summarizes the main findings of the two previous sub-sections.

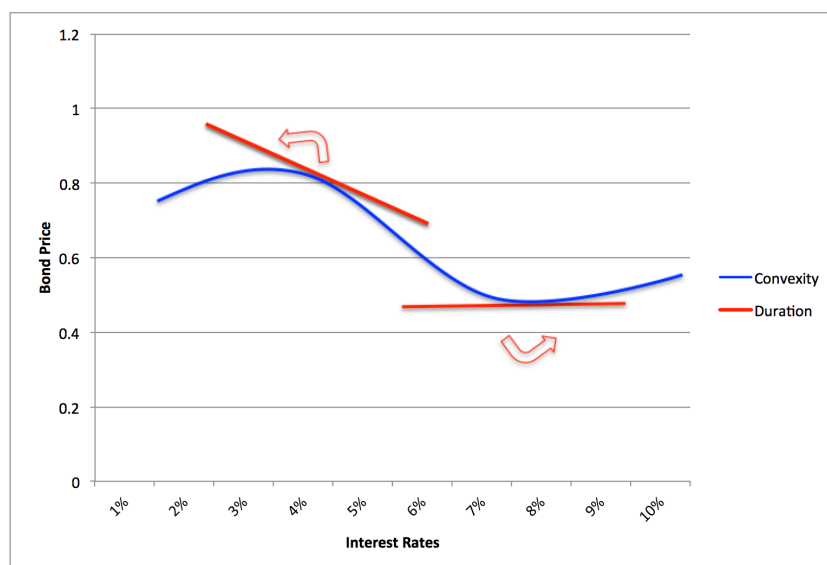
	Positive Gamma	Positive Convexity
Initial Strategy	Delta neutral	Duration neutral
Initial Positions	Long option Call + Short equity	Long bond + Short bond
Bullish Market Condition	Stock market price increases	Bond price increases
Impact on the portfolio	Long position appreciates more than Short one depreciates	Long Bond appreciates more than Short position depreciates
Keeping Portfolio Neutral	Decreasing the Long position or increasing the Short one	Decreasing the Long position or increasing the Short one

4. Why Dealing with Negative Convexity is Problematic?

A negative Convexity structure can be observed when you hold a callable bond, or a structured credit such as Asset Backed Securities. This structure can be problematic for traders since they will be Short when markets are bullish and Long when markets are bearish.

Imagine that you hold an ABS with an initial Duration of 5 years; this means that considering a given term structure, you expect all mortgages to be prepaid over the next 5 years [this includes anticipatively paid mortgages and those paid at maturity]. Imagine that the term structure shifts upwardly, the opportunity for mortgages to be refinanced anticipatively vanishes; the ABS portfolio will then see its average maturity increasing and the portfolio Duration passing, for instance, from 5 years to 7 years. This implies that if you want to keep a constant Duration, you are obliged to partially Short your Long position in ABS; you initially **are Long when markets are bearish, and you have to sell when prices are low** to maintain your portfolio Duration constant.

The chart below illustrates the negative Convexity effect:





5. Conclusion

Having a positive Gamma strategy when trading equity options can roughly be similarly interpreted as having a positive Convexity structure when trading Credit Instruments; this is what we exhibited within this working paper. We also proved that having a positive Gamma or a positive Convexity structure can be highly beneficial for traders from a P&L perspective, and that dealing with a negative Convexity could generate unexpected costs.