*At the beginning of a modeling process where parameters are derived from historical time-series, one of the first arising questions can be: what type of returns should I consider? Do I have to compute the arithmetic ones or the log ones? In Quantitative Finance, log returns are widely used; main reasons are listed in this working paper.*



## 1. Introduction

This paper is organised as follows: the first Section is dedicated to a straightforward comparison between the arithmetic returns and the log ones, Sections two to six are committed to the main advantages of log returns over the arithmetic ones. The final Section is dedicated to a brief conclusion.

## 2. Are Arithmetic Returns and Log Returns Interchangeable?

It is easy to exhibit that log returns tend to be smaller than arithmetic ones and are not perfectly interchangeable.

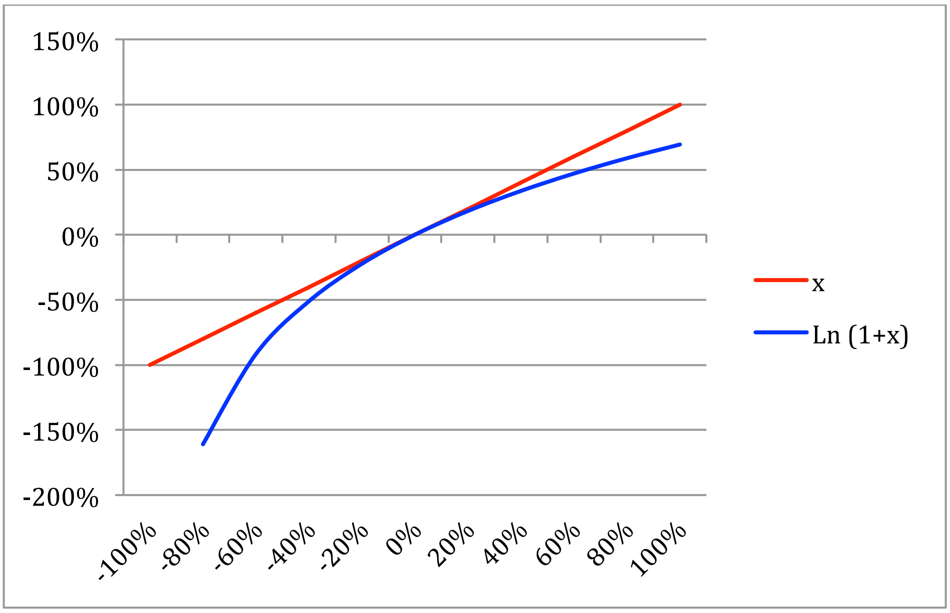
$$P_0 = 100$$

$$P_1 = 110$$

$$Arithmetic\ Returns = \left(\frac{P_1}{P_0}\right) - 1 = \frac{110}{100} - 1 = 10\%$$

$$Log\ Returns = ln\left(\frac{P_1}{P_0}\right) = \ln\left(\frac{110}{100}\right) = 9.53\%$$

The chart below exposes the relationship between arithmetic returns and log returns.




There are no one-to-one relationship between log returns and arithmetic ones; nevertheless you can note that the smaller the return, the more arithmetic and log returns tend to be similar.

### 3. Log Returns are not Impacted by the Compounding Frequency

Log returns tend to be considered as continuously compounded returns. This implies that the compounding frequency does not matter and different assets can easily be compared. As you can see below - and taking the example from Section 2 into consideration - the compounded return can directly be derived from the arithmetic one.

$$r_{compounded} = \ln(1 + r_{arithmetic}) = \ln(1.1) = 9.53\%$$

This feature is particularly useful when an investor wants to compute its final Wealth over a given period of time, or when it wants to compute the present value of a security or a portfolio of securities.

### 4. Log-Returns are Time-Additive

Assuming that log returns are continuously compounded returns, it can be demonstrated that log returns are time-additive. When considering a portfolio of multi-period assets, the n-period log return is equal to the sum of the single consecutive log returns. As you can see below:

$$P_0 = 100$$

$$P_1 = 110$$

$$P_2 = 120$$

$$Period\ 1 = \ln\left(\frac{110}{100}\right) = 9.53\%$$

$$Period\ 2 = \ln\left(\frac{120}{110}\right) = 8.70\%$$

Then

$$Period\ 1 + Period\ 2 = 8.70\% + 9.53\% = \ln\left(\frac{120}{100}\right) = 18.23\%$$

You can easily prove that this observation is not true for arithmetic returns.

## 5. With Small Returns, Log Returns are Approximately Equal to Arithmetic Ones

This has already been exposed with the chart of the Section 2. The postulate is the following:

$$r_{arithmetic} \approx \ln(1 + r_{arithmetic})\ for\ a\ small\ \ r_{arithmetic}$$

It can be demonstrated with the basic following example:

$$r_{arithmetic} = 0.5\%$$

$$\ln(\ 1 + 0.005) = 0.498\% \ \approx \ \ r_{arithmetic}$$

Please bear in mind that this cannot lead anyone to assume that the mean of returns derived from arithmetic returns is the same as the mean of returns obtained using logarithmic.

## 6. Log-Returns Can be Considered as Normally Distributed

I we assume that the price of a security follows a standard geometric Brownian motion, and is normally distributed; we can assume that Log returns of the security also are normally distributed. This is highly convenient to consider since many statistical models are based on the Gaussian normality (*eg.* the Black and Scholes model). This is also highly convenient when a shock price is added to time-series (*eg.* adding dividends to a net index).

## 7. Conclusion

If we only presented the main strengths of using log returns compared to arithmetic ones in this paper, please bear in mind that elements stated above do not hold in any situation, an that the modeller has to each time carefully consider if log returns are more appropriate than arithmetic ones.

## 8. References


- Hudson R., and Gregoriou A. (2010), "Calculating and Comparing Security Returns is harder than you think: A Comparison between Logarithmic and Simple Returns", Working Paper, *Hull University Business School*;
- Hughson E., Stutzer M., and Yung C. (2006) "The Misuse of Expected Returns", *Financial Analysts Journal*, 62, (Nov/Dec): 88-96.